Engineering Notes

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Convective Heat Flux in a Laser-Heated Thruster

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Introduction

A PROPULSION system based upon the interaction of laser radiation with a surface or a working fluid has been proposed.[1] This laser propulsion system combines, in principle, the very high specific impulse of nuclear or electric propulsion with the high thruster/weight ratio and safety of chemical rockets. Pirri et al.[2] have made preliminary assessments on the system and have shown that a thruster/power ratio of more than 10 dyn/W may be produced with pulsed 10.6 μ radiation and that a specific impulse of 1000 s can be achieved. More recently, the one-dimensional steady flow with laser energy addition has been examined for the application to the hypersonic wind tunnel and the laser-heated thruster.[3,4] Here we are primarily concerned with the convective heat flux in a laser-heated thruster.

Various aspects of a laser thruster with pure hydrogen as the flow medium have been examined.[4-6] Hydrogen has been chosen for high specific impulse. The laser beam is directed parallel to the flow. The plasma formed in the nozzle tends to propagate up the laser beam in the manner analogous to that of a laser-supported combustion wave, but is held stationary by the incoming flow. In the core calculations of Ref. 5 no heat loss (neigher radiative emission nor convection loss to the walls) was taken into account, and the average temperature of the hot plasma core was about 14,000 K. With the wall temperature range of about 300-1300 K (the melting temperature of copper), the transport properties vary significantly across the boundary layer under such extreme temperature variation (the $\rho\mu$, density-viscosity, product at the wall can be 10 times the freestream value). As shown in Fig. 1, the variation of $\rho\mu/\rho_e\mu_e$ is plotted against the normalized distance from the wall η which is zero at the wall and about five at the boundary-layer edge.[6] The subscript e indicates the conditions at the boundary-layer edge. The existing solutions with variable transport properties only cover the ratio of $\rho\mu$ product up to a factor of about 3. In addition, hydrogen at 3 atm is completely dissociated at about 5000 K. Hence, in order to estimate the convective heating to the wall, we have to solve the boundary-layer equations with variable transport properties.

In a laser-heated thruster, the core Reynolds number based on the nozzle throat diameter is approximately 2000. To estimate the heat transfer rate to the wall, we solve the laminar boundary-layer equations with local similarity approximation, which has long been used in exposing the principal physical features of boundary-layer phenomena and in providing a basis for calculating more complex, nonsimilar cases. For simplicity, we also assume that the gas phase is in equilibrium.

Now we proceed to assess the convective heat loss in the thruster examined in Ref. 5. From Ref. 5 we obtain the temperature and pressure distributions along the nozzle (from core calculation), Fig. 2. Knowing the freestream conditions of the boundary layer, one can obtain the similarity solutions and hence the heat flux which will indicate if it is necessary to couple the boundary losses directly to the core calculation. In addition, the effects of mass injection on the convective heat transfer across the boundary layer with such a large $\rho\mu$ gradient will be examined.

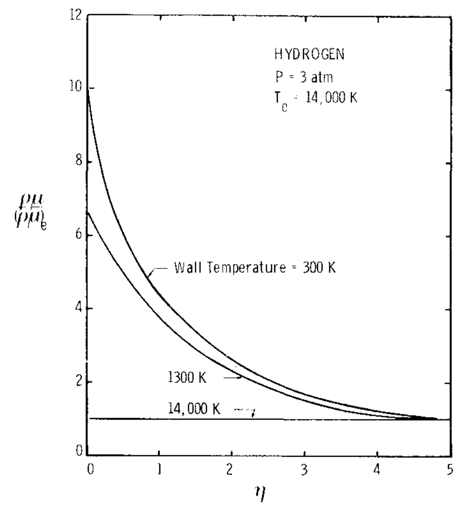

Fig. 1 Variation of density-viscosity product across the boundary layer.

Analysis

For simplicity, we assumed that Prandtl number $Pr = 1$ (the Prandtl number varies approximately from 0.7-1.2 across the equilibrium boundary layer) and $\rho \sim h^{-1}$ where ρ is the density and h is the enthalpy, and the two-dimensional similar boundary-layer equations are

$$[(\rho\mu/\rho_e\mu_e)f_{\eta\eta}]_\eta + ff_{\eta\eta} + \beta(g - f_\eta^2) = 0 \tag{1}$$

$$[(\rho\mu/\rho_e\mu_e)g_\eta]_\eta + fg_\eta = 0 \tag{2}$$

The boundary conditions are $f(0) = f_w$, $f_\eta(0) = 0$, $g(0) = g_w$, $f_\eta(\infty) = g(\infty) = 1$, where $f_\eta = u/u_e$ is the nondimensional velocity; $g = h_s/h_{se} = (h + u^2/2)/h_{se}$ is the stagnation ehthalpy ratio; and μ is the viscosity. The similarity variables are

$$\xi = \int_0^x \rho_e\mu_e u_e \mathrm{d}x \qquad \eta = (\rho_e u_e/\sqrt{2\xi}) \int_0^y \frac{\rho}{\rho_e}\mathrm{d}y$$

and the pressure gradient parameter is

$$\beta = \frac{2\xi}{u_e}\frac{\partial u_e}{\partial \xi}\frac{h_{se}}{h_e}$$

For reference, we take the wall temperature to be the melting point of copper, 1300 K. With the external conditions as

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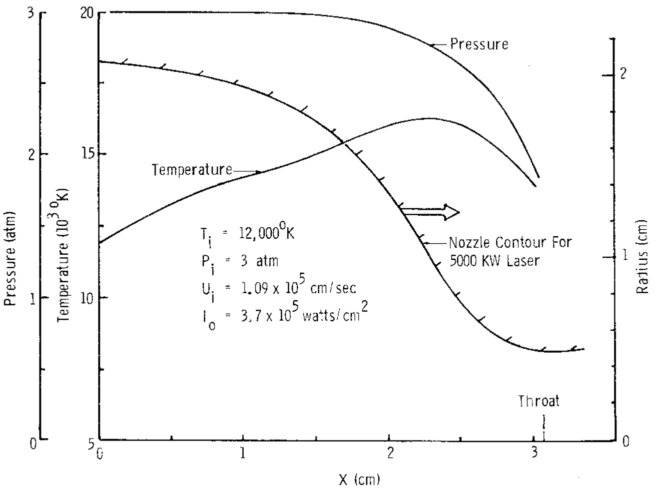

Fig.2 Temperature and pressure distributions along a particular nozzle contour.

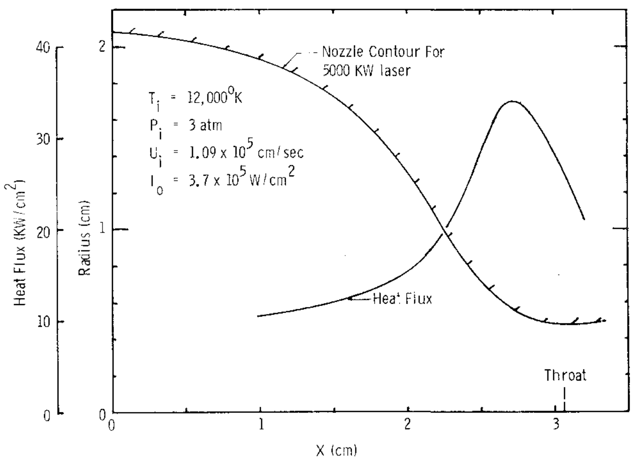

Fig. 3 Heat flux along the nozzle contour.

shown in Fig. 2, we can proceed to solve Eqs. (1) and (2) at each point along the nozzle wall and the heat flux at the wall can then be determined from the solution by the following expression:

$$q_w = (k_w \rho_w u_e h_{se} / c_{pw} \sqrt{2\xi}) g_\eta(0)$$

where k is the thermal conductivity, c_p is the specific heat at constant pressure, and subscript w indicates the wall condition.

Results and Discussion

In the absence of mass injection to protect the wall, i.e., $f_w = 0$, the similarity solutions for the nozzle, Fig. 2, have been obtained and the resultant convective heat flux distribution along a 1300 K wall is shown in Fig. 3.

In order to assess the effects of the convective heat loss to the wall, the size of the thruster, which depends on the laser power for a fixed laser intensity, has to be given. Here we will consider laser powers of both 100 and 5000 kW. As shown in Fig. 3, the convective heat flux varies from 10 kW/cm^2 to a peak of 34 kW/cm^2 near the throat. Integrating over the surface, one gets the total convective heat loss rate from

$$Q = \int_0^{x_{th}} 2\pi R q_w \mathrm{d}x$$

where R is the radius of the thruster and x_{th} is the throat location. Since the boundary-layer solution is singular at

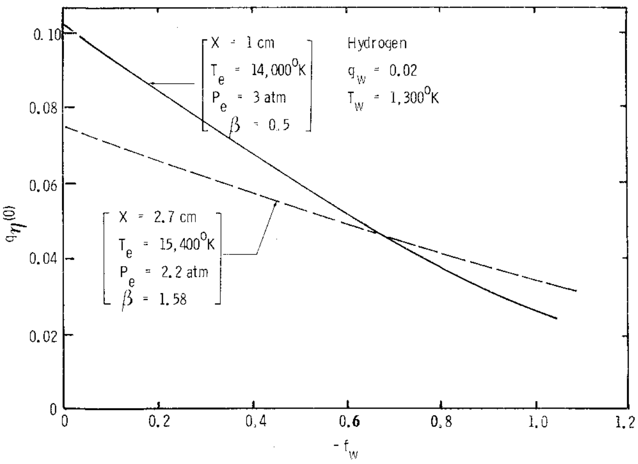

Fig. 4 Effect of normal hydrogen injection on heat flux parameter for hydrogen boundary layer.

$x = 0$, we only integrate from $x = 1.5$ cm to x_{th} and the values of Q for 100 and 5000 kW laser are 53 and 374 kW, respectively. It is evident that this uncoupled calculation produces unrealistic results for the 100 kW case, since the calculated heat loss rate is a large fraction of the total power. Such a loss rate would decrease the core temperature which, in turn, would decrease the heat loss. For this case, the convective heat loss must be coupled to the core calculation. For the 5000 kW case, however, the calculated loss rate is only 7.5% of the laser power, which is probably not a large enough loss to require coupling to the core calculation. Hence, the heat flux distribution shown in Fig. 3 is realistic for the 5000 kW case. This indicates the need for some protection of the thruster walls.

For cooling the thruster walls, forced convective water cooling can be used. This method of thruster wall cooling has been successfuly demonstrated with chemical rocket thruster chambers having maximum heat fluxes of about 16 kW/cm^2.[7] It is obvious that water cooling alone will not be sufficient to protect the walls in the present case.

One additional means of heat protection is the injection of cold hydrogen gas normal to the wall which alters the temperature profile and blocks some of the heat. Some sample calculations have been made to illustrate the effect of this method of reducing the heat flux. The calculation is made by introducing a nonzero value of f_w in the boundary conditions for Eqs. (1) and (2). The mass flux through the wall is related to f_w by

$$(\rho v)_w = -\rho_e \mu_e u_e f_w / \sqrt{2\xi} \tag{3}$$

The effects of mass injection have been assessed at two locations on the nozzle, i.e., $x = 1$ and 2.7 cm (the peak heat flux area). In Fig. 4, the curves of the heat flux parameter, $g_\eta(0)$ are given as a function of f_w for both cases. The left end of these curves is the zero injection or the solid wall case. It can be seen that $g_\eta(0)$ drops sharply as f_w decreases, which means the injection velocity increases, Eq. (3). If we look at effects of mass injection with $f_w = 1$, $g_\eta(0)$ at $x = 1$ cm is reduced by a factor of 4, leading to a heat flux of about 2.5 kW/cm^2 for an injection rate of 0.014 g/cm^2-s, from Eq. (3). At $x = 2.7$ cm, $g_\eta(0)$ is reduced by a factor of 2, yielding 17 kW/cm^2 with an injection rate of 0.017 g/cm^2-s. To grasp the magnitude of this rate of injection in the 5000 kW/cm^2 case, one may compare it to the mass flow in the core which is 4.5 g/s. Assuming an average injection rate of 0.015 g/cm^2-s over the entire thruster wall, one finds that the total injected mass would be 0.49 g/s, or only 11% of the core mass flow. However, large reductions in heat flux are only needed in the limited regions of high heating. Hence, it appears that it is possible to protect the thruster walls by the combination of mass injection into the boundary layer and forced convection water cooling. In addition, a third configuration of having a buffer gas between the plasma and the wall can also be used to reduce the heat loss. However, considerable additional calculations would be needed for an engineering analysis of the proper amount and distribution of injection for efficient heat protection in a laser-heated thruster.

Acknowledgments

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References

[1]Kantrowitz, A.R., "Propulsion To Orbit By Ground-Based Laser," *Astronautics and Aeronautics,* Vol. 10, May 1972, pp. 74-76.

[2]Pirri, A.N. and Weiss, R.F., "Laser Propulsion," AIAA Paper 72-719, Boston, Mass., 1972.

[3]Hertzberg, A., Johnston, E.W., and Ahlstrom, H.G., "Photon Generators and Engines for Laser Power Transmission," AIAA Paper 71-106, New York, N.Y., 1971.

[4]Wu, P.K. and Pirri, A.N., "Stability of Laser Heated Flows," *AIAA Journal,* Vol. 14, March 1976, pp. 390-392.

[5]Wu, P.K., "Real Gas Effect in a Laser Heated Thruster," *AIAA Journal,* Vol. 14, Dec. 1976, pp. 1766-1768.

[6]Wu, P.K., "Similarity Solution of the Boundary Layer Equations for Laser Heated Flows," *AIAA Journal,* Vol. 14, Nov. 1976, pp. 1659-1661.

[7]Shoji, J.M. and Larson, V.R., "Performance and Heat Transfer Characteristics of the Laser-Heated Rocket-A Future Space Transportation System," Paper 76-1044, AIAA International Electric Propulsion Conference, Nov. 1976.

Temperature Sensitivity and Erosive Burning

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REDUCTION of the effects of initial propellant temperature on the performance of solid rocket motors has been an elusive goal of propellant formulators for some time. Recently, nozzleless rocket motor tests have shown temperature sensitivities substantially below those for nozzled motors.[1] This suggests that paths to reduced motor temperature sensitivity may exist outside the propellant formulators domain. For nozzleless motors, the observed effects should be traceable to propellant deflection, erosive burning, and velocity profile effects. The purpose of this Note is to explore the effect of erosive burning on motor temperature sensitivity.

Erosive burning phenomena are complex and, currently, the cause is disputed. King[2] assumes that the primary effect is flame bending by the velocity gradient near the burning surface whereas Lengelle,[3] Beddini,[4] and Condon and Osborn[5] assume that the primary effect is due to eddy transport in the gas phase reaction zone. However, qualitative results from either school are similar. This is not unexpected. Turbulence near the burning surface is required to produce a steep velocity gradient there and Reynolds analogy is roughly valid for the combustion gases. For the purposes herein (scouting to see if the effect of erosion is beneficial, detrimental, or null) the simplest possible formulation consistent with the phenomena should be adequate. The Lenoir-Robillard relation[6] is simple, of closed form, contains the major physical elements, and has demonstrated its ability to correlate data. Unfortunately, a central aspect of that theory, the concept of erosion being dependent upon convective heat transfer from the "core gas" to the burning surface, has been shown to be fallacious by King.[2] However, this central dogma is more illusion than substance because of the interdependence of transport phenomena when the Prandtl and Schmidt numbers are close to unity. An energy balance at the burning surface gives

$$r\rho_s[c_s(T_s-T_i)-Q_s] \sim (\lambda_g+\overline{\rho_g c_g \epsilon})(T_f-T_s)/L \qquad (1)$$

where r is the burning rate, ρ is density, c is specific heat, λ is thermal conductivity, T is temperature, Q_s is the energy release in the surface reaction, ϵ is turbulent diffusivity, and subscripts g, s, f, i, refer to gas, surface, flame, and initial conditions, respectively. For most composite solid propellants the activation energy of the surface reaction is large (O[30,000 kcal/mole]) so that T_s is substantially constant. Since,

$$r_o\rho_s[c_s(T_s-T_i)-Q_s] \sim \lambda_g(T_f-T_s)/L \qquad (2)$$

for burning without crossflow and $r=r_o+r_e$

$$r_e\rho_s[c_s(T_s-T_i)-Q_s] \sim \overline{\rho_g c_g \epsilon}(T_f-T_s)/L \qquad (3)$$

Lengelle[3] has shown that the characteristic thickness of the gas phase reaction zone L is independent of crossflow. Therefore, the variable in the turbulent problem is turbulent transport. For turbulent, convective heat transfer

$$h=\overline{\rho_g c_g \epsilon}/L \qquad (4)$$

Thus, making use of turbulent heat transfer to evaluate the turbulent transport

$$r_e=h(T_f-T_s)/\{\rho_s[c_s(T_f-T_s)-Q_s]\} \qquad (5)$$

Following Lenoir and Robillard

$$h=0.0288\, Gc_g Re_x^{-0.2} Pr^{-0.667} \exp(-\beta r\rho_s/G) \qquad (6)$$

where G is the crossflow mass flux, Re_x is the length x Reynolds number, β is a parameter, and Pr is the Prandtl number.

Thus, the erosive burning rate r_e is

$$r_e=\alpha^*[(T_f-T_s)/\{\rho_s[c_s(T_s-T_i)-Q_s]\}]G^{0.8}x^{-0.2} \times \exp(-\beta r\rho_s/G) \qquad (7)$$

where α^* is a parameter dependent upon gas phase fluid properties alone. Consequently, the Lenoir-Robillard relation is reached without heat transfer from a core flow. In short, Lenoir and Robillard have the "correct" equation but the wrong concept. The experimental work of Yamada et al.[7] also implies this.

To accomplish the stated task, burning rate is first expressed in terms of stationary and erosive terms. Then the erosive part is related to the crossflow. Finally, the crossflow is related to the static properties (p, M, T_i) defining the situation. Then by employing the proper definitions, the desired results may be achieved.

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